NONUNIFORM IMAGE RECONSTRUCTION USING MULTILEVEL SURFACE INTERPOLATION

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ABSTRACT

This paper describes a fast algorithm for nonuniform image reconstruction. A multiresolution approach is formulated to compute a C^2-continuous surface through a set of irregularly spaced samples. The algorithm makes use of a coarse-to-fine hierarchy of control lattices to generate a sequence of surfaces whose sum approaches the desired interpolating surface. Experimental results demonstrate that high fidelity reconstruction is possible from a selected set of sparse and irregular samples.

1. INTRODUCTION

Nonuniform image reconstruction refers to the problem of fitting a smooth function through a nonuniform, or scattered, distribution of image samples. This subject is closely related to the general problem of scattered data interpolation, which is a subject is of practical importance in many science and engineering fields, where data is often measured or generated at sparse and irregular positions. The goal of interpolation is to reconstruct an underlying function (e.g., surface) that may be evaluated at any desired set of positions. This serves to smoothly propagate the information associated with the scattered data onto all positions in the domain.

Despite a flurry of activity in this area, nonuniform image reconstruction remains a difficult and computationally expensive problem. The vast literature devoted to this subject documents various approaches, many of which suffer from limitations in smoothness, time complexity, or allowable data distributions [2]. A trend in recent algorithms has been the use of hierarchical, or multiresolution, filtering to extend onto all positions the information known only at the sparse and irregular samples. Burt proposed hierarchical polynomial fit filtering to yield a multiresolution set of low-pass filtered images that can be combined to form a smooth surface passing through the original data [1]. Mitchell proposed multistage filtering to handle highly variable sample density [6]. In that work, weighted-average filters are repeatedly applied with ever-narrowing low-pass cutoff until the proper bandwidth for the display is reached.

This paper introduces a fast algorithm for constructing a C^2-continuous interpolation function from arbitrary scattered data. The algorithm applies an effective approximation technique to a hierarchy of control lattices to generate a sequence of functions whose sum approaches the desired interpolation function. The work is based on the multilevel approximation technique presented in [4] for image morphing, where multilevel B-splines were used to propagate user-specified values at scattered features across the image. Full details of the scattered data interpolation algorithm presented in this paper, including pseudocode, may be found in [5].

2. SURFACE APPROXIMATION

Let $\Omega = \{(x, y)|0 \leq x < m, 0 \leq y < n\}$ be a rectangular domain in the xy-plane, where x, y are real-valued coordinates and m, n are integers. Consider a set of scattered image samples $P = \{(x_c, y_c, z_c)\}$, where z_c is the image intensity of a sample lying at position (x_c, y_c) in domain Ω. We formulate the reconstructed image to be a uniform bicubic B-spline surface passing through P. The B-spline surface is defined by a control lattice Φ overlaid on domain Ω. Without loss of generality, we assume that Φ is an $(m+3) \times (n+3)$ lattice which spans the integer grid in Ω.

Let ϕ_{ij} be the value of the ij-th control point on lattice Φ, located at (i, j) for $i = -1, 0, ..., m+1$ and $j = -1, 0, ..., n+1$. The reconstructed function (surface) f is defined in terms of these control points by

$$f(x, y) = \sum_{k=0}^{3} \sum_{l=0}^{3} B_k(s) B_l(t) \phi_{(i+k)(j+l)}, \qquad (1)$$

where $i = \lfloor x \rfloor - 1$, $j = \lfloor y \rfloor - 1$, $s = x - \lfloor x \rfloor$, and $t = y - \lfloor y \rfloor$. B_k and B_l are uniform cubic B-spline basis functions. They serve to weigh the contribution of each control point to $f(x, y)$ based on its distance to (x, y). With this formulation, the problem of deriving function f is reduced to solving for the control points in Φ that best approximate the scattered data in P.

To determine the unknown control lattice Φ, we first consider one data point (x_c, y_c, z_c) in P. From Eq. (1), we know that function value $f(x_c, y_c)$ relates to the sixteen control points in the neighborhood of (x_c, y_c). Without loss of generality, we may assume that (x_c, y_c) are chosen such that $i = j = 0$ in Eq. (1). Then, control points ϕ_{kl}, for $k, l = 0, 1, 2, 3$, must satisfy

$$z_c = f(x_c, y_c) = \sum_{k=0}^{3} \sum_{l=0}^{3} w_{kl} \phi_{kl}, \tag{2}$$

where $w_{kl} = B_k(s) B_l(t)$ and $s = x_c - 1$, $t = y_c - 1$. There are many values for the ϕ_{kl}'s that satisfy Eq. (2). We choose one in the least-squared sense that minimizes $\sum_{k=0}^{3} \sum_{l=0}^{3} \phi_{kl}^2$. The solution is calculated as [3]:

$$\phi_{kl} = \frac{w_{kl} z_c}{\sum_{a=0}^{3} \sum_{b=0}^{3} w_{ab}^2}. \tag{3}$$

Now, we consider all the data points in P. For each data point, Eq. (3) can be used to determine the set of 4×4 control points in its neighborhood. These neighborhoods may overlap for sufficiently close data points. They may assign different values to several shared control points. In general, we resolve multiple assignments to a control point ϕ by considering the data points in its 4×4 neighborhood. Only these points may influence the value of ϕ by Eq. (3). For each of these points, Eq. (3) gives ϕ_{ij} a different value ϕ_c. To compromise among the values, ϕ_{ij} is chosen to minimize error $e(\phi_{ij}) = \sum_c (w_c \phi_{ij} - w_c \phi_c)^2$. The term $(w_c \phi_{ij} - w_c \phi_c)$ is the difference between the real and expected contributions of ϕ_{ij} to function f at (x_c, y_c). The least-squared solution is

$$\phi_{ij} = \frac{\sum_c w_c^2 \phi_c}{\sum_c w_c^2}. \tag{4}$$

We shall refer to this B-spline approximation process as the BA algorithm.

The density of control lattice Φ overlaid on domain Ω directly affects the shape of approximation function f. As Φ becomes coarser, more data points lie in the neighborhood of a control point, thereby influencing its value. This causes many data points to be blended together to yield a smoother shape for f at the expense of approximation accuracy. However, as Φ becomes finer, the influence of a data point is limited to smaller neighborhoods. This enables P to be more closely approximated, although f will tend to contain local peaks near the data points.

3. MULTILEVEL SURFACE INTERPOLATION

A tradeoff exists between the shape smoothness and accuracy of the approximation function generated by the BA algorithm. In this section, we present a multiresolution algorithm to circumvent this tradeoff. The algorithm makes use of a hierarchy of control lattices to generate a sequence of functions f_k whose sum approaches the desired approximation function. In the sequence, a function from a coarse lattice provides a rough approximation, which is further refined in accuracy by functions derived from finer lattices.

Consider a hierarchy of control lattices, Φ_0, Φ_1, ..., Φ_h, overlaid on domain Ω. We assume that the spacing between control points for the coarsest lattice Φ_0 is given and that the spacing is halved from one lattice to the next. Therefore, if Φ_k is $(m_k + 3) \times (n_k + 3)$ control points, the next finer lattice Φ_{k+1} will have $(2m_k + 3) \times (2n_k + 3)$ control points. The position of the ij-th control point in Φ_k coincides with that of the $(2i, 2j)$-th control point in Φ_{k+1}.

We begin the multilevel interpolation process by applying the BA algorithm to P with the coarsest control lattice Φ_0. The resulting function f_0 serves as a smooth initial approximation that possibly leaves large discrepancies at the data points in P. In particular, f_0 leaves a deviation $\Delta^1 z_c = z_c - f_0(x_c, y_c)$ for each point (x_c, y_c, z_c) in P. The next finer control lattice Φ_1 is then used to obtain function f_1 that approximates the difference $P_1 = \{(x_c, y_c, \Delta^1 z_c)\}$. Then, the sum $f_0 + f_1$ yields a smaller deviation $\Delta^2 z_c = z_c - f_0(x_c, y_c) - f_1(x_c, y_c)$ for each point (x_c, y_c, z_c) in P. We shall refer to this multilevel B-spline approximation process as the MBA algorithm.

For a level k in the hierarchy, we derive function f_k by using control lattice Φ_k to approximate data $P_k = \{(x_c, y_c, \Delta^k z_c)\}$, where $\Delta^k z_c = z_c - \sum_{i=0}^{k-1} f_i(x_c, y_c) = \Delta^{k-1} z_c - f_{k-1}(x_c, y_c)$, and $\Delta^0 z_c = z_c$. This process starts from the coarsest lattice Φ_0 and continues incrementally to the finest lattice Φ_h. The final interpolation function f is defined as the sum of functions f_k, i.e., $f = \sum_{k=0}^{h} f_k$. Note that only the coarsest lattice Φ_0 is applied to the original data P to derive the global shape of function f. All successively finer lattices serve to approximate and remove the residual error. In this manner, we have an incremental solution for function f that yields a smooth interpolation to P. Note that interpolation is actually achieved once sufficiently fine lattices are introduced to remove all residuals.

4. EXAMPLES

Consider a set of nonuniform samples taken from an image. Reconstruction refers to the interpolation necessary to recover the image from its samples. A robust solution to this problem is of great practical importance in image compression. Significant data reduction is possible by representing a full image with only a few scattered samples. We now address this problem by interpreting a grayscale image as a surface, where the value of each pixel represents its height. Image reconstruction from nonuniform samples is then cast into a surface fitting framework that can be solved with the MBA algorithm.

Consider the image in Fig. 1(a). To derive a set of nonuniform samples, we first apply an edge detector to the image and threshold the result to identify edges. These edge contours consist of an important set of pixels which capture the visual details of the image. Due to their rather arbitrary definition and distribution, edges alone are not sufficient to recover the image. Therefore, we also sample the image on a coarse regular grid to properly reconstruct the parts of the image where there are no nearby edge points. The positions of the sampled pixels are shown in Fig. 1(b). For the purpose of this example, we used a uniform sampling rate of one sample per six pixels along both directions. In order to demonstrate the role of both sets of samples (edges and low-res uniform samples), we applied the MBA algorithm to both sets separately to reconstruct the images shown in Figs. 1(c) and 1(d), respectively. Note that neither of these two images is adequate. They both, however, complement each other, i.e., one accurately resolves the high frequencies and the other only resolves low frequencies. When the MBA algorithm is applied to all samples depicted in Fig. 1(b), the surface in Fig. 1(e) is fitted. The reconstructed image that corresponds to this surface is shown in Fig. 1(f).

Large data reduction is achieved here. The dimensions of the original image in Fig. 1(a) is 360×243. The number of sample points in Fig. 1(b) is 7301, of which 4984 lie upon the edge contours. Therefore, sample points constitute only 8.3% of the total number of pixels in the original image. Fig. 1(f) demonstrates that the MBA algorithm can adequately reconstruct an image from a proper set of samples. The original and reconstructed images are visually indistinguishable.

5. CONCLUSIONS

This paper has presented a fast algorithm for nonuniform image reconstruction. The algorithm is based on multilevel surface interpolation. A B-spline control lattice is efficiently determined by minimizing a local approximation error for each control point. The resulting C^2-continuous function passes near densely distributed data points and interpolates isolated points. However, a tradeoff exists between the shape smoothness and approximation accuracy of the function, depending on the control lattice density.

Multilevel B-spline approximation was introduced to circumvent this tradeoff. The algorithm makes use of a hierarchy of control lattices to generate a sequence of functions whose sum approaches the desired approximation function. In the sequence, a function from the coarsest control lattice provides an initial estimate, which is further refined in accuracy by functions derived at finer levels.

We also presented a sufficient condition for multilevel B-spline approximation to generate an interpolation function through scattered data points. Interpolation is achieved when the control point spacing in the finest lattice becomes sufficiently small relative to the data distribution.

The scattered data interpolation technique introduced here was demonstrated for image reconstruction from nonuniform samples. We have demonstrated that high fidelity image reconstruction is possible from a selected set of sparse and irregular samples. The algorithm may play a significant role in data compression. Future work will address techniques to uniquely determine a minimal set of samples necessary to achieve high quality reconstruction within a specified error tolerance.

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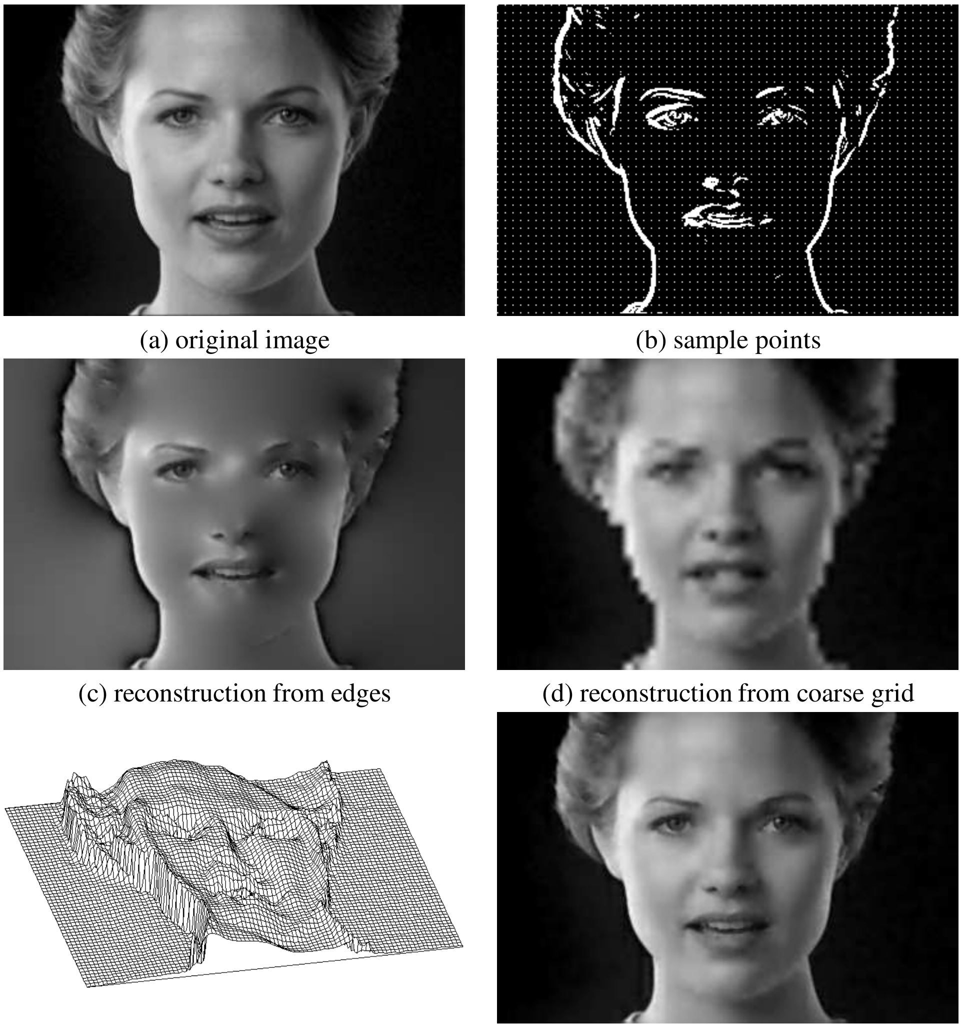

(a) original image (b) sample points

(c) reconstruction from edges (d) reconstruction from coarse grid

(e) reconstructed surface (f) reconstructed image

Figure 1: Image reconstruction example