Image Warping: A Review

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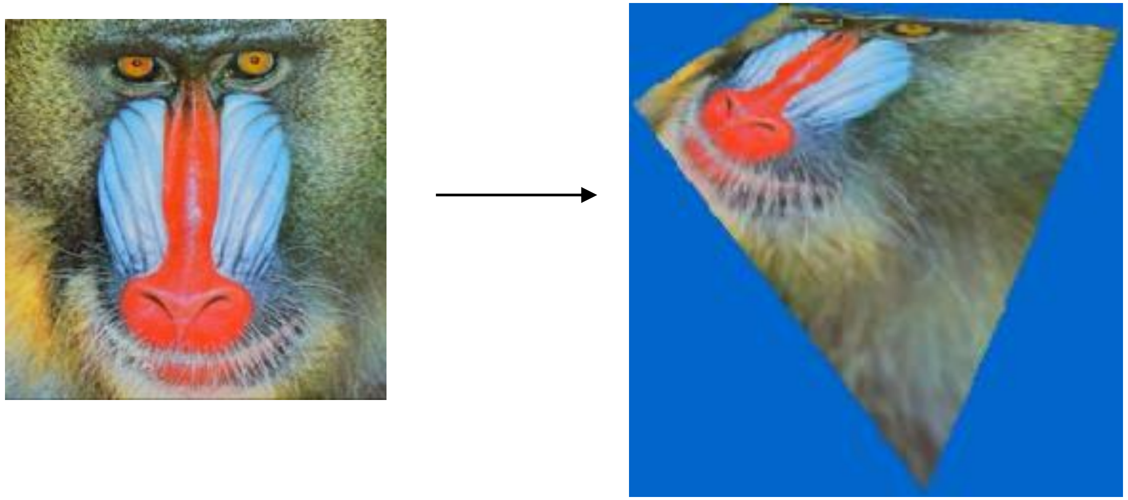
Objectives

- In this lecture we review digital image warping:
  - Geometric transformations
  - Forward inverse mapping
  - Sampling
  - Image reconstruction
  - Interpolation kernels
  - Separable transforms
  - Fant’s resampling algorithm

Definition

- Image warping deals with the geometric transformation of digital images.



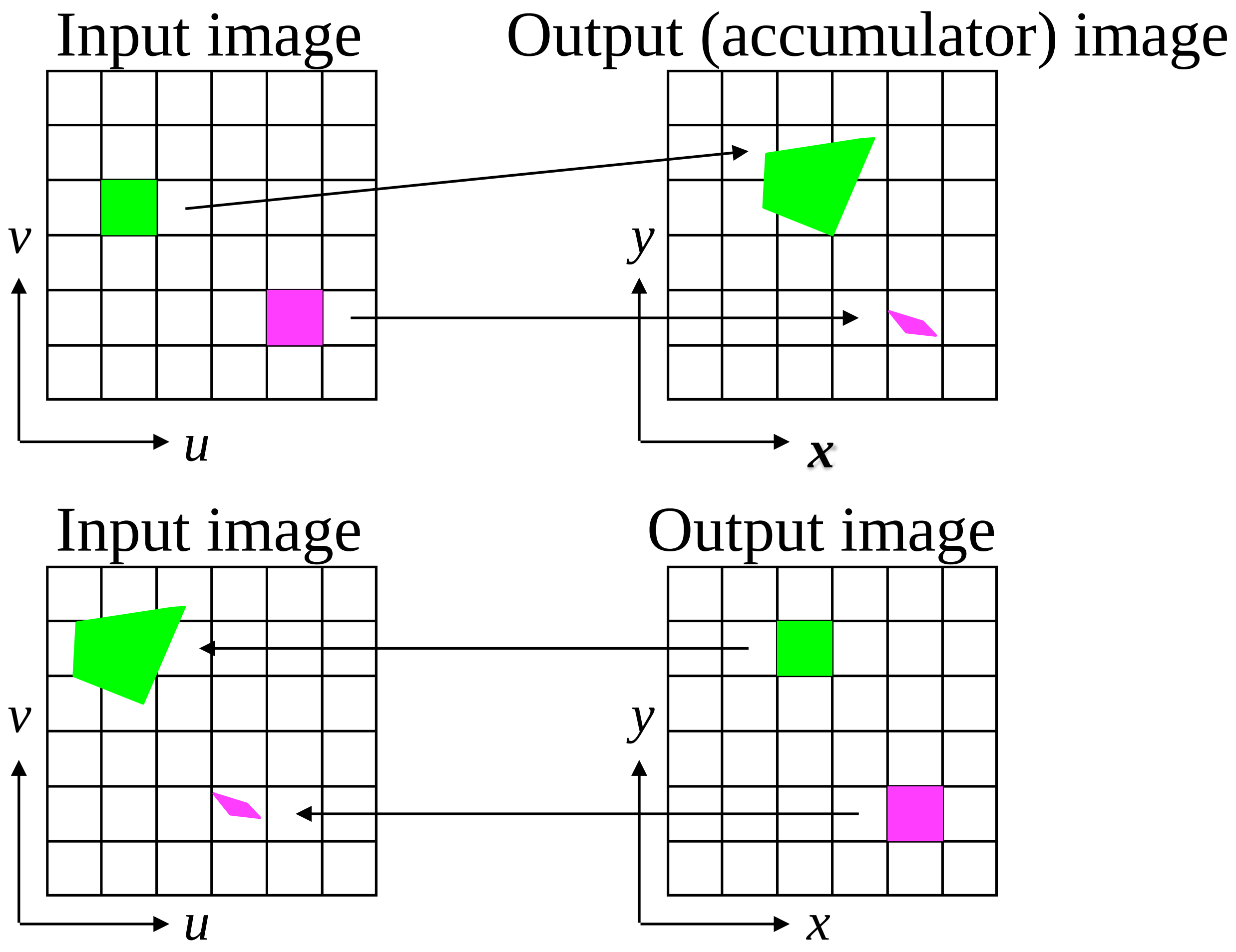
Geometric Transformations

- Affine
- Perspective
- Bilinear
- Polynomial
- Splines
- Elastic (local deformations)

Spatial Transformations

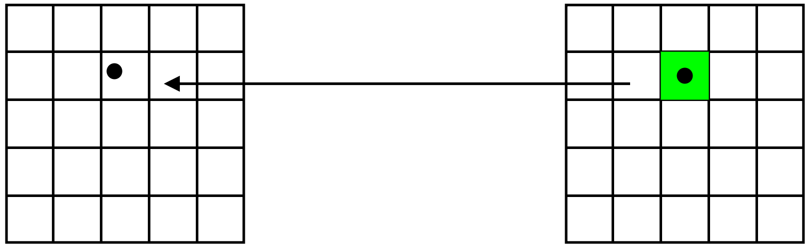
- Forward Mapping $[x, y] = [X(u, v), Y(u, v)]$
- Inverse Mapping $[u, v] = [U(x, y), V(x, y)]$

Forward / Inverse Mapping

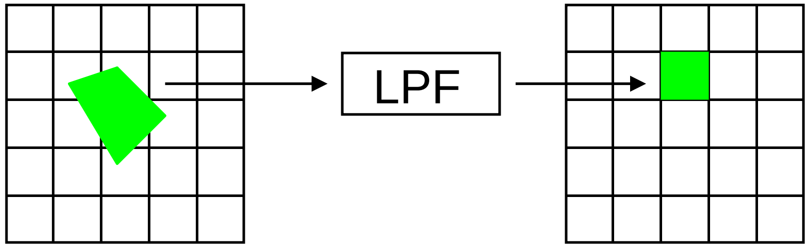


Sampling

- Point Sampling



- Area Sampling

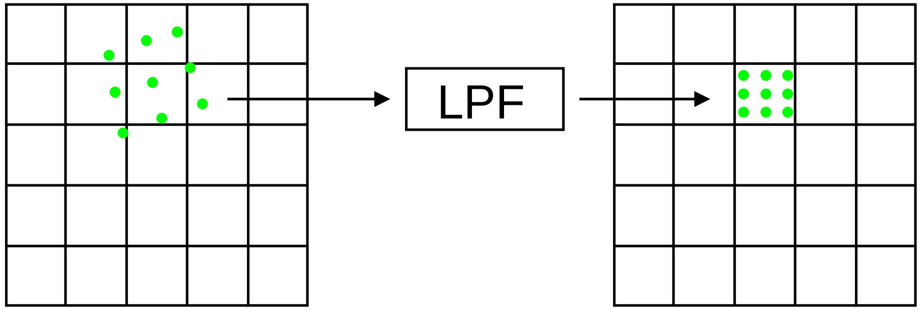


Area Sampling

- Treats pixels as finite areas
- Avoids aliasing (undersampling) artifacts
- Approximated by supersampling

Supersampling

- Average of projected subpixels



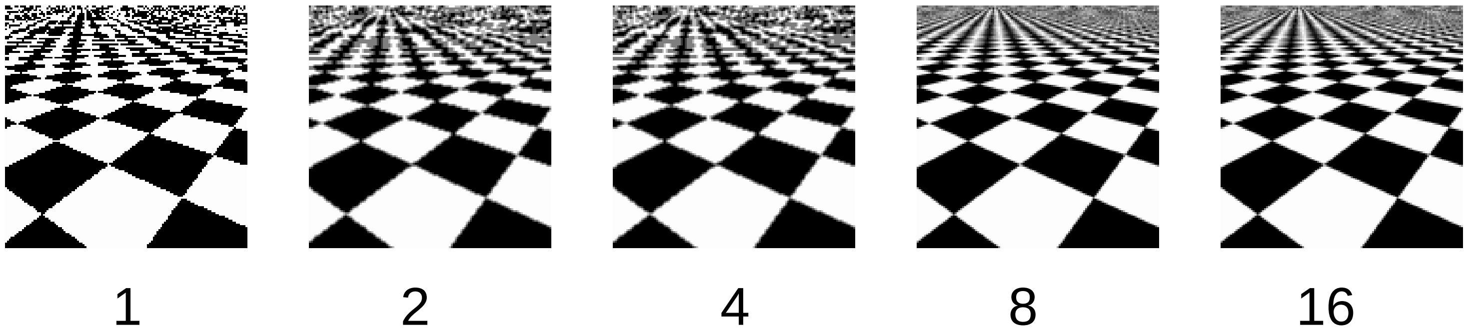
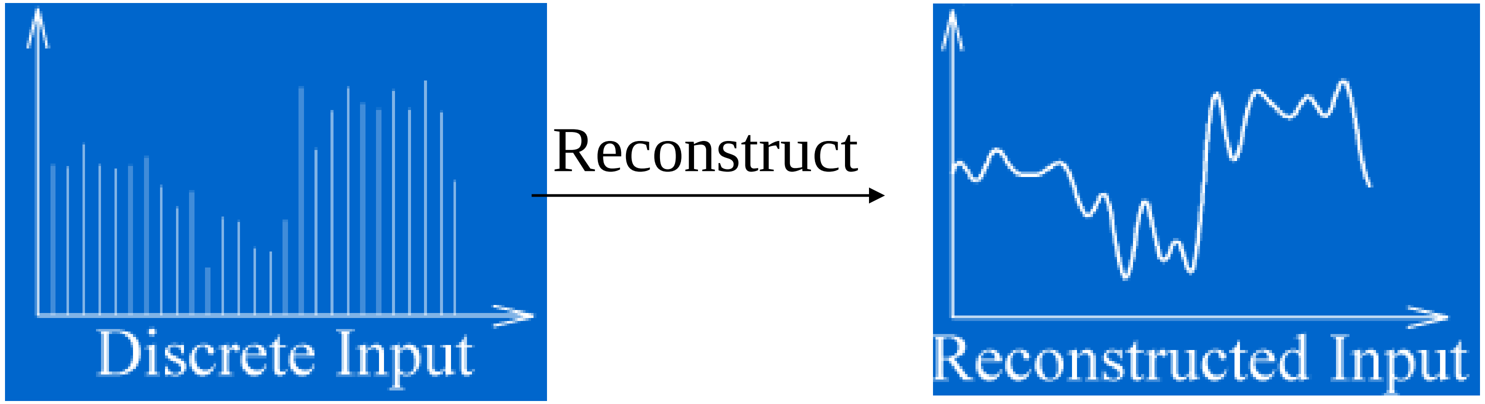


Image Reconstruction

- Pixel values are known at integer positions
- Samples can project to real-valued positions
- How do we evaluate the image values at these real-valued positions? Reconstruction

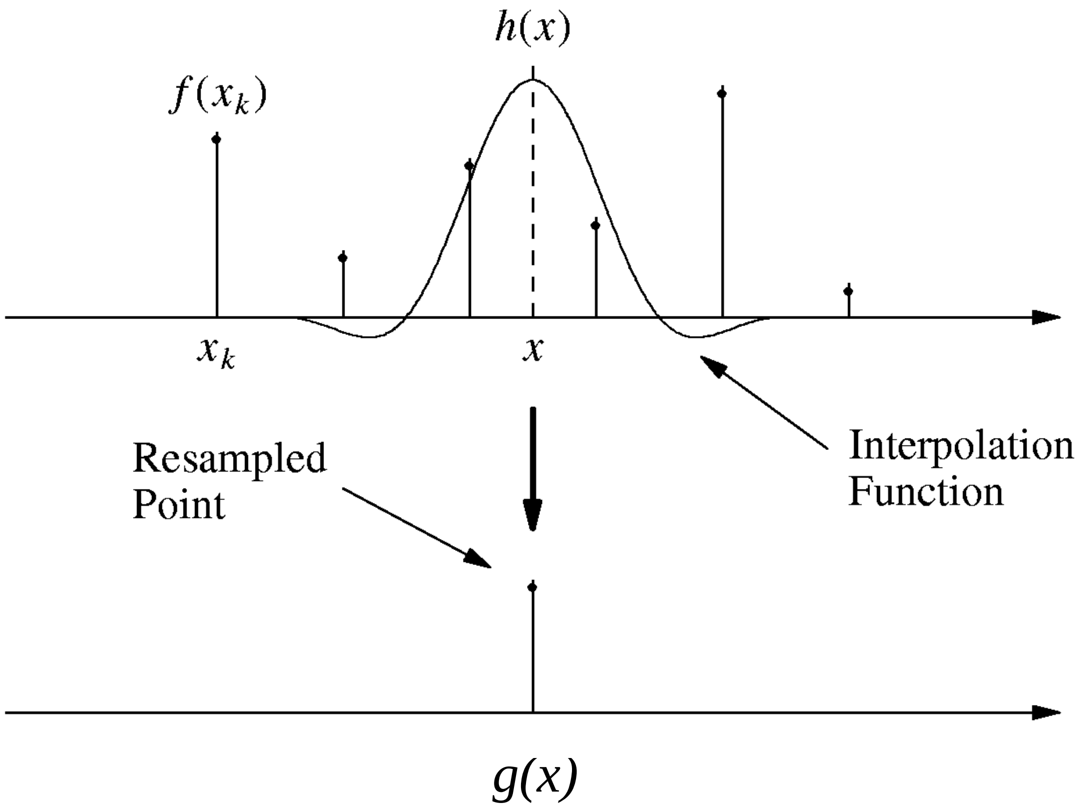


Interpolation

- Reconstruction interpolates the input
- In practice, interpolation is performed at points of interest only, not entire function
- Interpolation is achieved by convolution

Convolution

$$g(x) = \sum_{k=0}^{N} f(x_k) h(x - x_k)$$



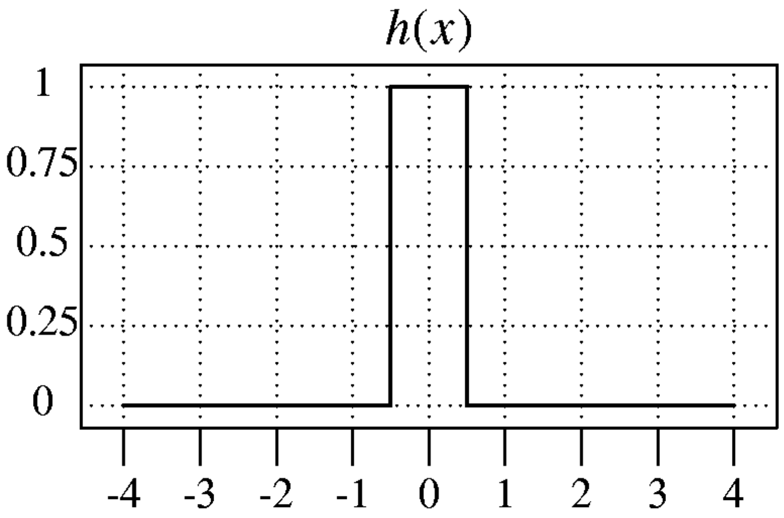
Interpolation Functions

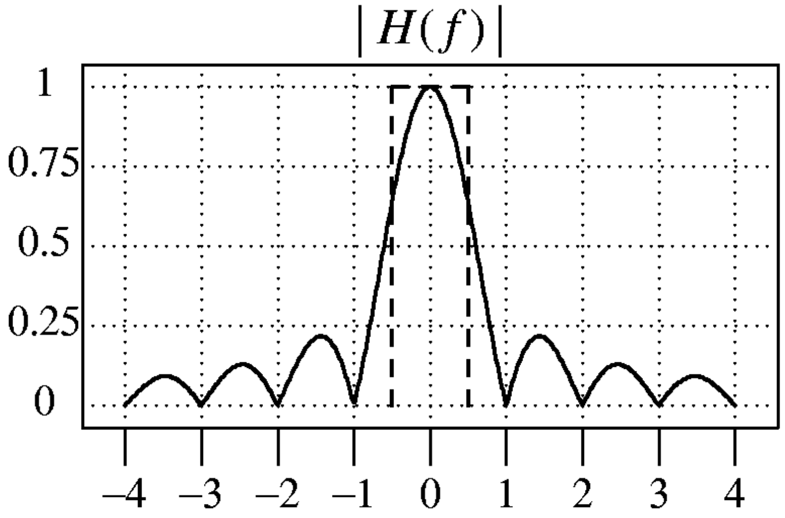
Interpolation functions/kernels include:

- Box filter
- Triangle filter
- Cubic convolution
- Windowed sinc functions

Box Filter

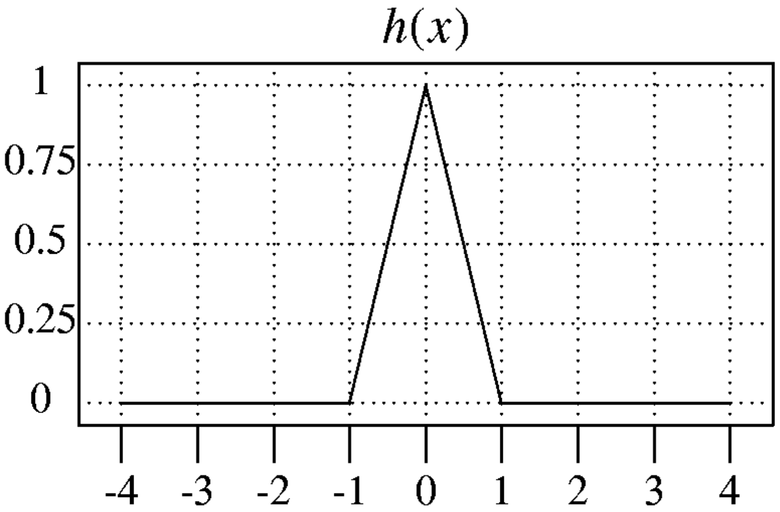
- Nearest neighbor interpolation
- Blocky artifacts may occur

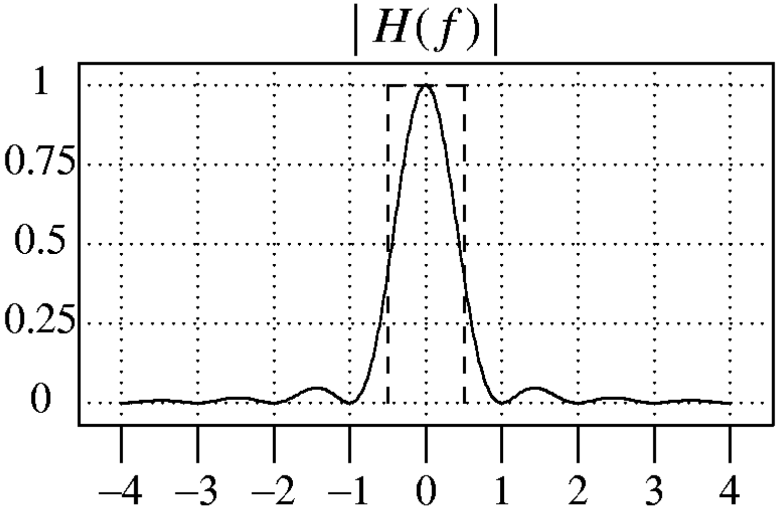




Triangle Filter

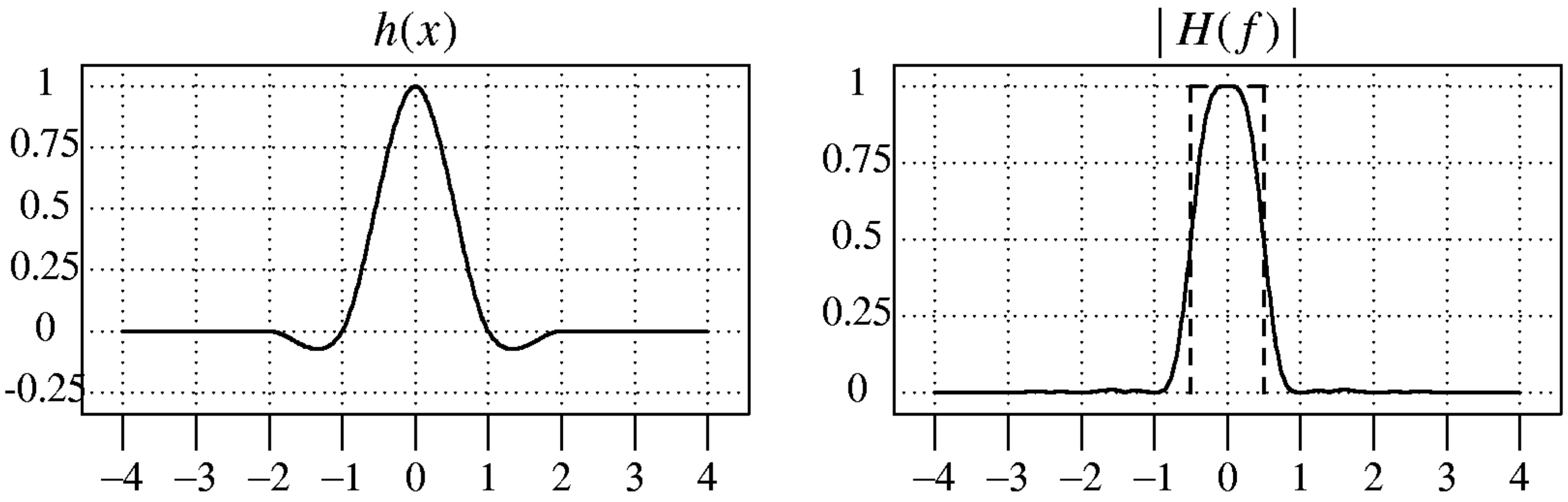
- Linear interpolation
- Popular for use with small deformations





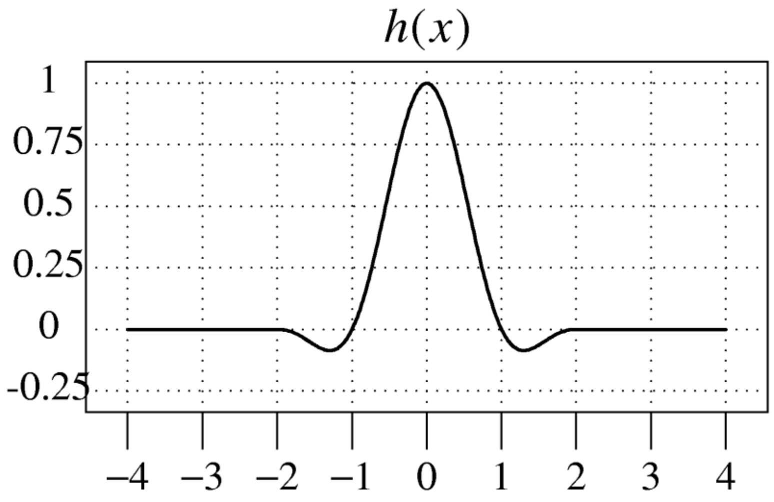
Cubic Convolution

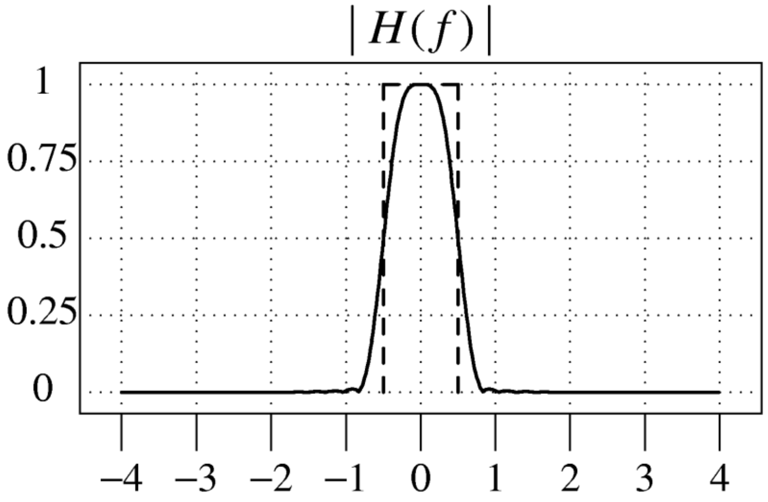
- Local cubic interpolation algorithm
- Advanced feature in digital cameras



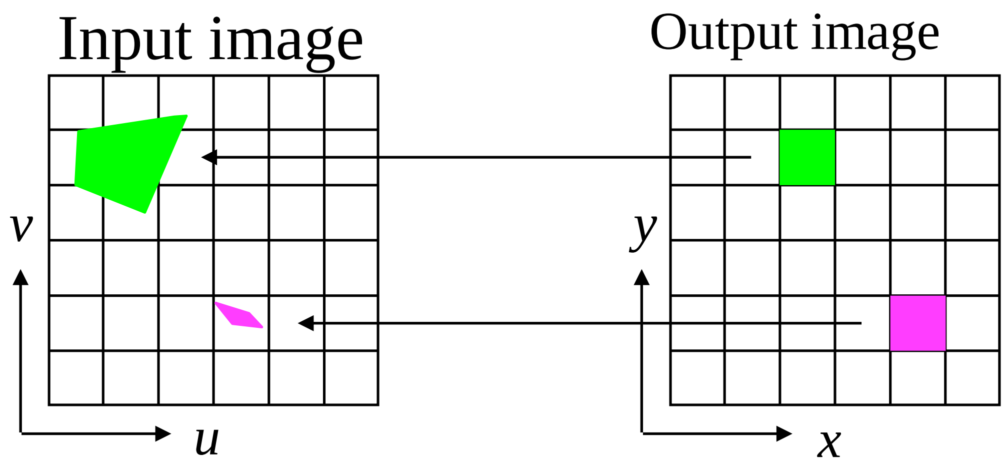
Windowed Sinc Function

- Smoothly tapered ideal sinc function



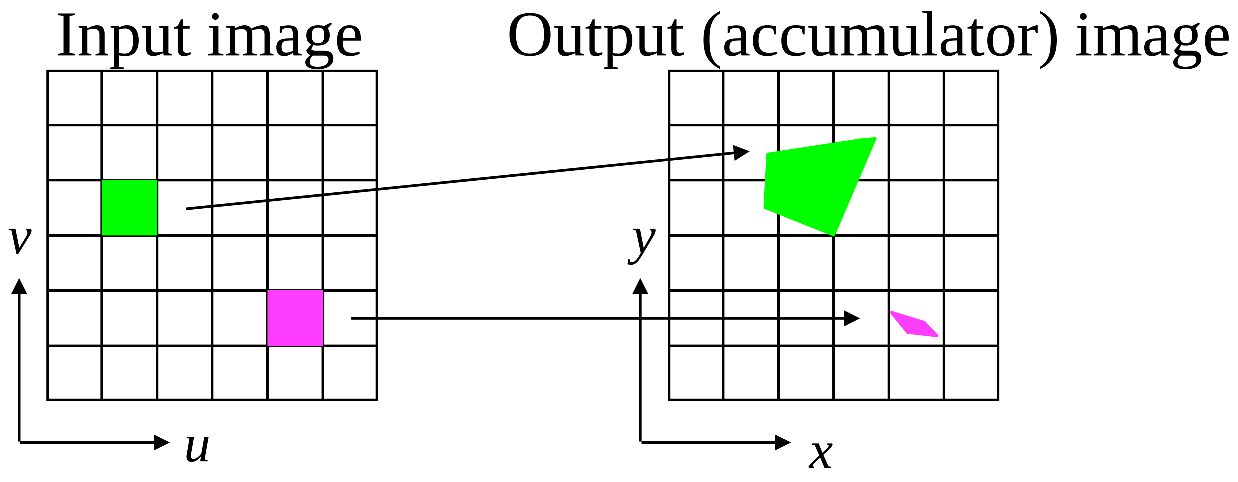


Inverse Mapping



- Visit output in scanline order
- Supersampling approximates area sampling
- Popular in computer graphics

Forward Mapping



- Visit input in scanline order
- Use output accumulator array
- 2D antialiasing is difficult
- Separable transforms facilitate efficient solution

Separable Transforms

$$[X(u,v), Y(u,v)] = F(u,v) \circ G(x,v)$$

- $F(u, v)$ is a row-preserving transformation that maps all input points to their final column positions, i.e., $[x, v]$.

- $G(x, v)$ is a column-preserving transformation that maps the $[x, v]$ points to their final row positions, i.e., $[x, y]$.

Catmull-Smith Algorithm

- **First pass**

  Maps image $S(u,v)$ into intermediate image $I(x,v)$

$$I(x,v) = S(X(u,v), v)$$

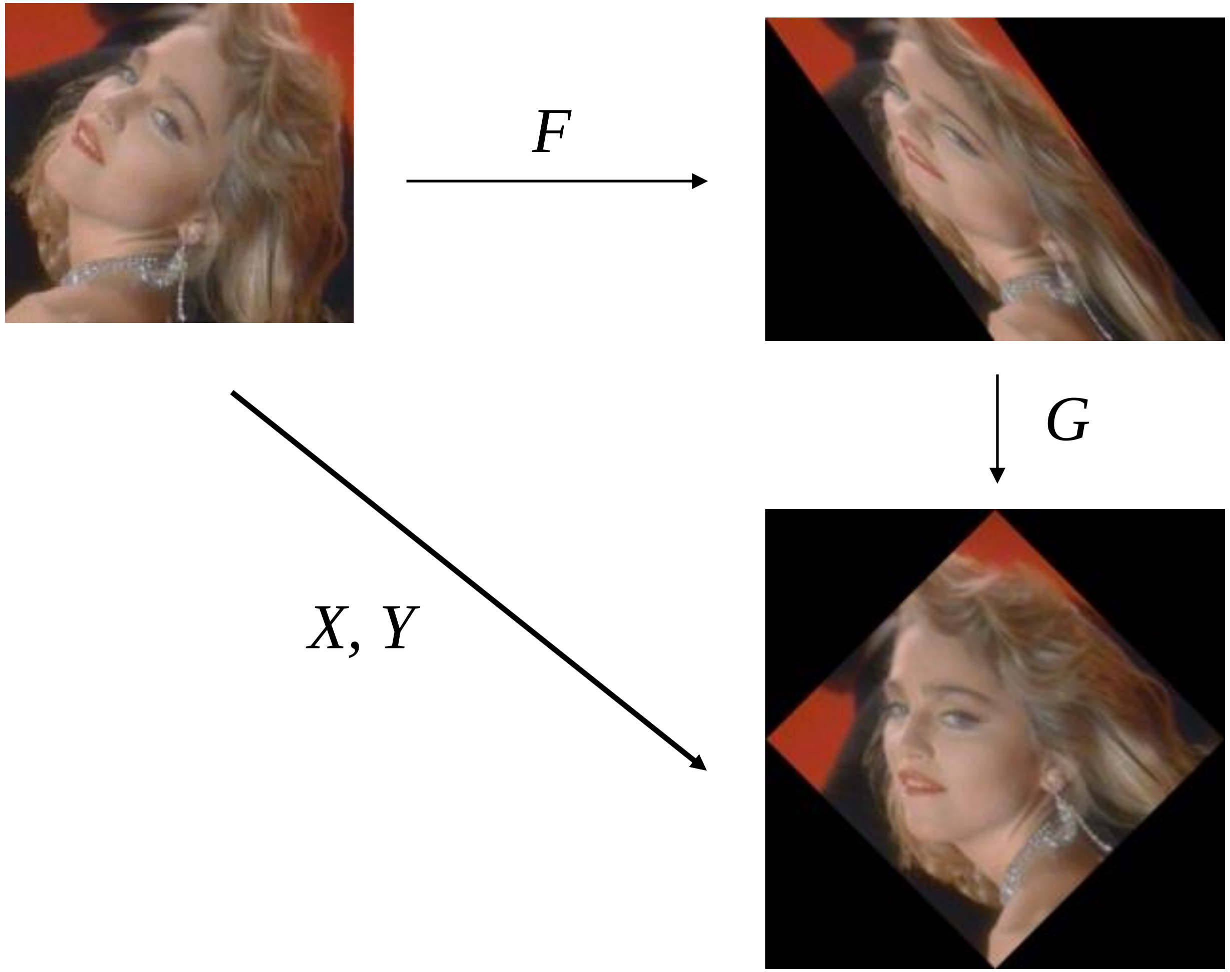
- **Second pass**

  Maps $I(x,v)$ into target image $T(x,y)$

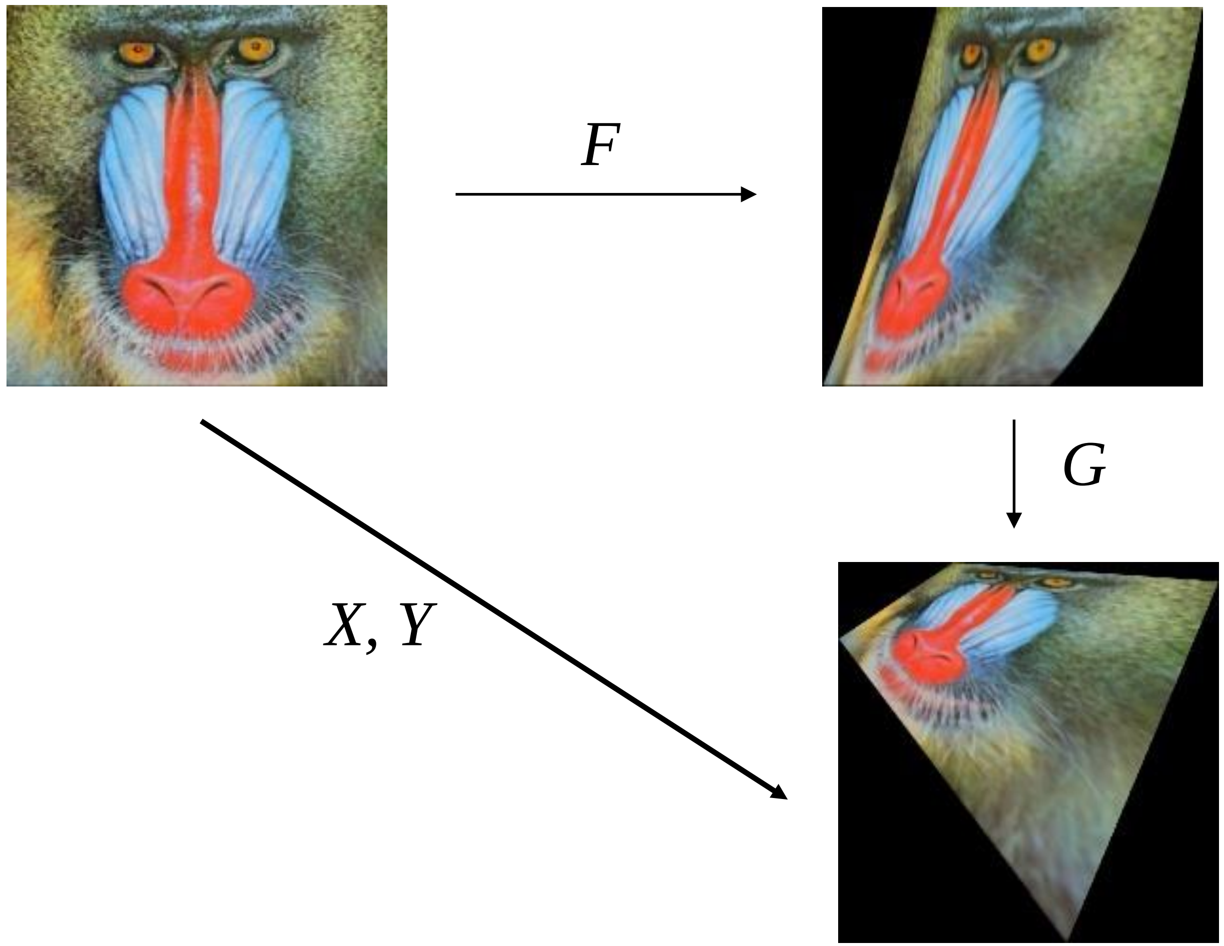
$$T(x,y) = I(x, Y(H_x(v), v))$$

  where H_x is the solution to $x=X(u,v)$ for u

2-Pass Rotation



2-Pass Perspective



Fant's Algorithm

- Forward mapping intensity resampling
- Scanline order in input *and* output
- Amenable to hardware implementation

Fant's algorithm: Example (1)

$XLUT$	.6	2.3	3.2	3.3	3.9
$YLUT$	100	106	115	120	
I	100	106	92	90	
$YLUT_x$	100	101	105	113	
I_x	40	101	106	82	

Fant's algorithm: Example (2)

$$I_x(0) = (100)((.4)) = 40$$

$$I_x(1) = \left[(100)\left[1 - \frac{.4}{1.7}\right] + (106)\left[\frac{.4}{1.7}\right]\right]((1)) = 101$$

$$I_x(2) = \left[(100)\left[1 - \frac{1.4}{1.7}\right] + (106)\left[\frac{1.4}{1.7}\right]\right]((.3)) + (106)((.7)) = 106$$

$$I_x(3) = \left[(106)\left[1 - \frac{.7}{.9}\right] + (92)\left[\frac{.7}{.9}\right]\right]((.2)) + (92)((.1)) + (90)((.6)) = 82$$

Bibliography

- Catmull, E. and A.R. Smith, "*3-D Transformations of Images in Scanline Order*," Proc. Siggraph '80, pp. 279-285, 1980.
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- Wolberg, George, *Digital Image Warping*, IEEE Computer Society Press, Los Alamitos, CA 1990.