CSc 220: Algorithms
Homework 9
Due in Class on Thursday November 19

Return the homework written on sheet(s) of paper with your name and CSc220 written at the top of each sheet. Please staple multiple sheets together. Remember that collaboration is allowed, but that you must write the solution on your own. Also you must acknowledge all collaborators and all sources (other than the textbook) in your solutions. Each problem is worth 10 points.

Problem 1: In class we saw how to compute all-pairs shortest paths in a graph $G$ by computing $W^{n-1}$ where $W$ is the (weighted) adjacency matrix and $W^k$ is defined as $W \otimes \ldots \otimes W$ ($k$ times) where $\otimes$ is the row-by-column matrix "multiplication" operation we saw in class (and described by pseudocode on page 624 of the textbook). Prove that this multiplication operation is associative (which is necessary to utilize the repeated squaring technique to compute $W^{n-1}$ with $O(\log n)$ multiplications).

Problem 2: What happens when you run Johnson’s algorithm on a graph that has all positive weights on its edges? What are the values of the functions $h(v)$ and $\hat{w}(e)$ for any node $v$ and edge $e$?

Problem 3: In class we saw how to compute the transitive closure of a graph using a variation of the Floyd-Warshall algorithm that uses Boolean values and operations instead of the integer operation over the weights. Show how to compute the transitive closure using the same type of variation in the "matrix multiplication" algorithm to compute all-pairs shortest paths.