CSc 220: Algorithms
Homework 5
Due in Class on Thursday October 13

Return the homework written on sheet(s) of paper with your name and CSc220 written at the top of each sheet. Please staple multiple sheets together. Remember that collaboration is allowed, but that you must write the solution on your own. Also you must acknowledge all collaborators and all sources (other than the textbook) in your solutions. Each problem is worth 10 points.

**Problem 1:** In class we saw how to use AVL trees to maintain a dynamic set on which you need to perform the following operations: **INSERT, DELETE, SEARCH, SUCCESSOR.** All these operations run in $O(\log n)$ time on a tree with $n$ elements.

We now want to add the procedure **RANK.** When we call Rank$(x, i)$ it returns the $i^{th}$ element in the tree rooted at $x$.

Show how to implement this operation in $O(\log n)$ time on a modified version of AVL trees.

*Hint:* Add one additional field to each node of the AVL tree that will help you answer the rank operator efficiently. Make sure that you can maintain this field efficiently as you insert and delete elements into the tree.

**Problem 2:** The **Brocard point** of a triangle ABC is the point P in the triangle chosen so that $\angle PAB = \angle PBC = \angle PCA$ as described in the figure below.

The common angle is called the Brocard angle. The largest possible Brocard angle is $\frac{\pi}{6}$ for the case of an equilateral triangle.

Given the coordinates of the vertices as input, how would you compute the coordinates of the Brocard point?

**Problem 3:** City College is holding a fundraiser at the XYZ corporation. Each employee $x$ at XYZ as pledged to contribute $c_x$ to City College if invited to the fundraiser, otherwise he/she will contribute nothing.

The employees at XYZ are organized according to a hierarchical structure which we can represent as a tree. The XYZ CEO is at the top and the children of each node are its direct subordinates.

To avoid fights at the party and make sure that people can talk and drink freely, City College does not want to invite an employee and his/her direct supervisor to the party.

Your job is to come up with a guest list that satisfies the above constraints and maximizes the amount of donations that City College will receive at the party.

*Hint:* Use dynamic programming. Faster algorithms will get better scores on the homework.