CSc 220: Algorithms
Homework 2
Due in Class on Thursday September 14

Return the homework written on sheet(s) of paper with your name and CSc220 written at the top of each sheet. Please staple multiple sheets together. Remember that collaboration is allowed, but that you must write the solution on your own. Also you must acknowledge all collaborators and all sources (other than the textbook) in your solutions. Each problem is worth 10 points.

Problem 1: A different way to randomize QUICK-SORT is to use the deterministic version of QUICK-SORT over a 'randomized' array, according to the following pseudo-code

PERMUTE-QUICK-SORT(A)
  $B \longleftarrow$ RANDOM-PERMUTE(A);
  RETUR N QUICK-SORT(B)

- Give a sufficient condition on the procedure RANDOM-PERMUTE that will make PERMUTE-QUICK-SORT run in $O(n \log n)$ steps. [6pts]
- Consider the following procedure

  SHIFT-PERMUTE(A)
    $n \leftarrow |A|$;
    $s \leftarrow$ RANDOM$(1, n)$;
    FOR $i = 1$ TO n
      $j \leftarrow s + i \bmod n$;
      $B[j] \leftarrow A[i]$;
    RETURN B

  What is the expected running time of PERMUTE-QUICK-SORT if you use SHIFT-PERMUTE in place of RANDOM-PERMUTE? [4pts]

Problem 2: Let a and b be two n bit numbers (assume for simplicity that n is a power of 2).

- Describe the "grade school" algorithm to multiply a and b and show that it requires $O(n^2)$ steps; [2pts]
- Describe a divide-and-conquer algorithm with an asymptotically faster running time. [8pts]

Problem 3: You are given n samples of a chemical compound. While they look identical, some of them have in fact been contaminated. You have a testing machine that given two samples can detect if they are the same or not. You also know that most of the samples (a majority of them) are identical. Find one of those identical samples making no more than n tests with your machine (a.k.a. comparisons).