CSc 220: Algorithms
Homework 1
Due in Class on Thursday September 7

Return the homework written on sheet(s) of paper with your name and CSc220 written at the top of each sheet. Please staple multiple sheets together. Remember that collaboration is allowed, but that you must write the solution on your own. Also you must acknowledge all collaborators and all sources (other than the textbook) in your solutions. Each problem is worth 10 points.

Problem 1: Assume that k, ϵ are constants with $k \geq 1$ and $0 < \epsilon < 1$. Also log denotes the logarithm in base 2, and ln the natural logarithm in base e. State which among $A = O(B)$, $A = \Omega(B)$ and $A = \Theta(B)$ is correct for each pair of function below. Justify your answer. 2 points per question.

- $A = \log^2 n$ and $B = n^{1/100}$
- $A = n^k$ and $B = n^{\frac{\ln n}{\log n}}$
- $A = 5^n$ and $B = 4^n$
- $A = n^{\log^2 n}$ and $B = 2^{\log^3 n}$
- $A = 3^n$ and $B = 2^{n^2}$

Problem 2: For each of the following recurrences: (i) describe what kind of "divide and conquer" algorithm would give rise to such a recurrence; (ii) give asymptotic upper and lower bounds on $T(n)$. Make your bounds as tight as possible and justify your answer. Assume $T(n)$ is a constant for $n \leq 2$.

- $T(n) = 3T(n/3) + n \log n$ [3points]
- $T(n) = \sqrt{n}T(\sqrt{n}) + n$ [3 points]
- $T(n) = 2T(n/2) + \frac{n}{\log n}$ [4 points]

Problem 3: Given a set A of n positive integers and another interger t, describe an algorithm that determines whether or not there exists two elements in A such that their product is exactly t. Prove that your algorithm is correct and analyze its running time. Full credit will be given to the fastest algorithm.