Encoding Information

Encoding numbers

Rational Numbers: Integers (m,n) and $n \neq 0$

N bits

-2^{N-1}	2^{N-2}	2^{N-3}	....	. . .	2^{-1}	2^{-2}	2^{-3}

Sign bit

Fixed point

Moving the location of fixed point we determine the integer range and fraction range.

$$11000101 = -2^{4} + 2^{3} + 2^{-1} + 2^{-3} = -16+8+0.5+0.125 = -7.325$$

Fixed point

$$11000101 = -2^{1} + 2^{0} + 2^{-4} + 2^{-6} = -2 + 1 + 0.125 + 0.03125 = -0.84375$$

Fixed point

Rational Number Program

Objective
Your assignment is to implement a program that will be capable of adding, subtracting, multiplying and dividing rational numbers.

Example

If you enter two rational numbers $\frac{1}{2}$ and $\frac{1}{2}$ you should get the following results.

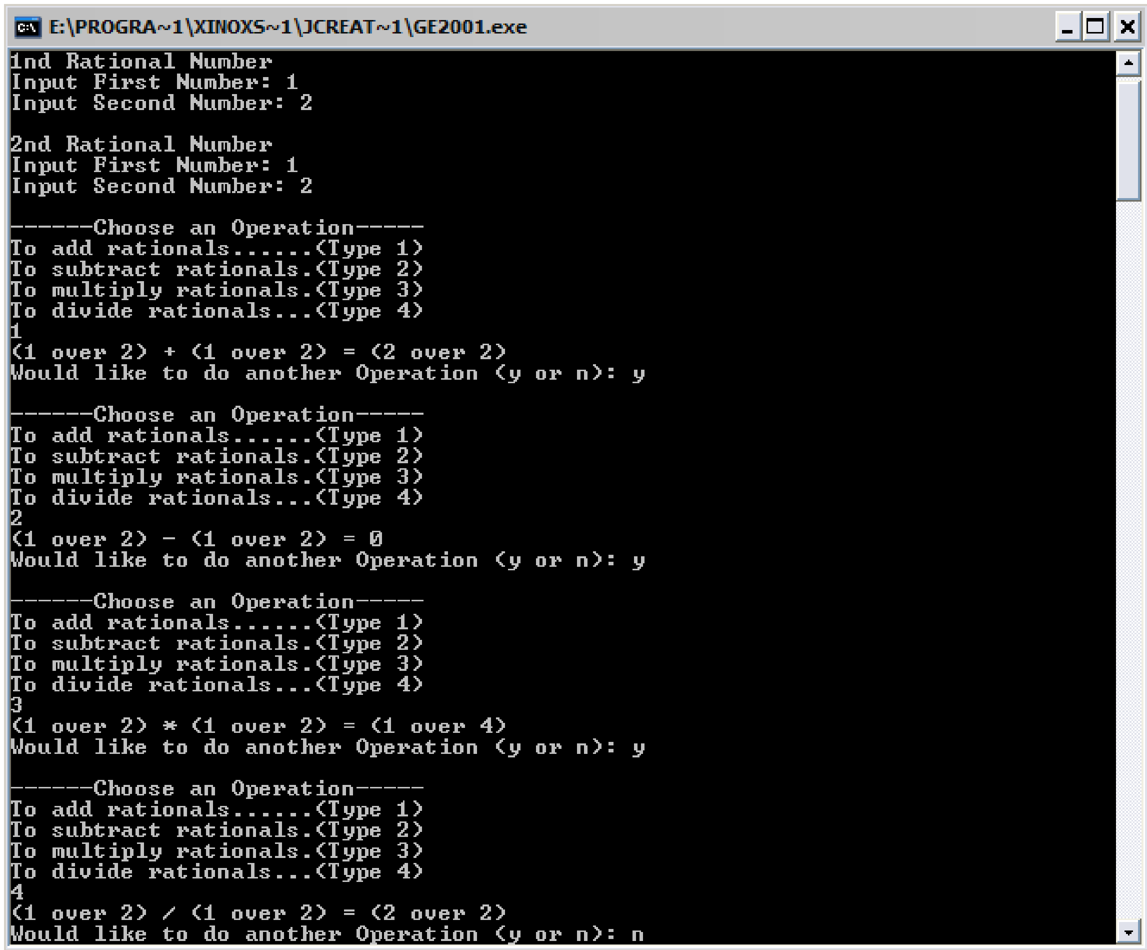
$$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=1$$

$$\frac{1}{2}-\frac{1}{2}=0$$

$$\frac{1}{2}*\frac{1}{2}=\frac{1}{4}$$

$$\frac{1}{2}/\frac{1}{2}=\frac{2}{2}=1$$

After typing the same rational numbers into the program we get results that should look similar to the results above. An example is shown below.



```
E:\PROGRA~1\XINOXS~1\JCREAT~1\GE2001.exe
1nd Rational Number
Input First Number: 1
Input Second Number: 2

2nd Rational Number
Input First Number: 1
Input Second Number: 2

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
1
(1 over 2) + (1 over 2) = (2 over 2)
Would like to do another Operation (y or n): y

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
2
(1 over 2) - (1 over 2) = 0
Would like to do another Operation (y or n): y

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
3
(1 over 2) * (1 over 2) = (1 over 4)
Would like to do another Operation (y or n): y

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
4
(1 over 2) / (1 over 2) = (2 over 2)
Would like to do another Operation (y or n): n
```

Now let's try another set of rational numbers to check if the program truly works for this lets choose the rational numbers $\frac{1}{3}$ and $\frac{1}{2}$. Here the results should be.

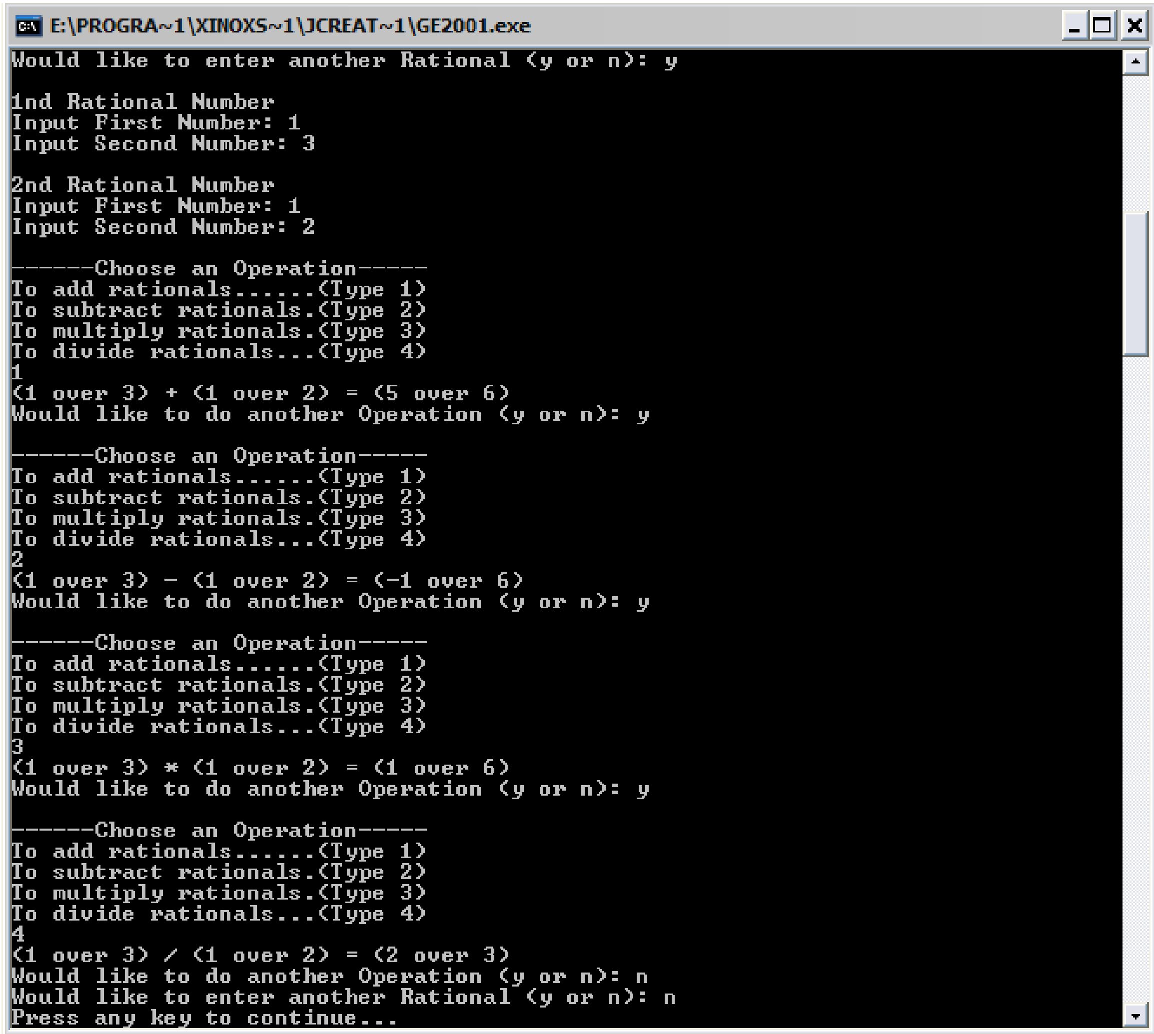
$$\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$$

$$\frac{1}{3}-\frac{1}{2}=-\frac{1}{6}$$

$$\frac{1}{3}*\frac{1}{2}=\frac{1}{6}$$

$$\frac{1}{3}/\frac{1}{2}=\frac{2}{3}$$

After typing these rational numbers into the program we get results that should look similar to the results above.


E:\PROGRA~1\XINOXS~1\JCREAT~1\GE2001.exe
Would like to enter another Rational (y or n): y

1nd Rational Number
Input First Number: 1
Input Second Number: 3

2nd Rational Number
Input First Number: 1
Input Second Number: 2

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
1
(1 over 3) + (1 over 2) = (5 over 6)
Would like to do another Operation (y or n): y

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
2
(1 over 3) - (1 over 2) = (-1 over 6)
Would like to do another Operation (y or n): y

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
3
(1 over 3) * (1 over 2) = (1 over 6)
Would like to do another Operation (y or n): y

------Choose an Operation-----
To add rationals......(Type 1)
To subtract rationals.(Type 2)
To multiply rationals.(Type 3)
To divide rationals...(Type 4)
4
(1 over 3) / (1 over 2) = (2 over 3)
Would like to do another Operation (y or n): n
Would like to enter another Rational (y or n): n
Press any key to continue...

Encoding Information

Encoding describes the process of assigning representations to information

Digital Sound

8-BIT may encode one amplitude (level)out of 256 per one sample.

16-BIT Code may represent an amplitude (one from 65k) of one DIGITAL SOUND sample

SAMPLING RATE = Number of samples per one second.

6 x 16-BIT = 96 bits may encode one sample of a 6 channel DIGITAL SOUND

Number of bits required to encode 1 sec of digital sound =
(Number of bits per sample) x (Number of samples per one sec)

How much memory do you need to store 1 hour of stereo music using
16 bit quantization and sampling rate =40,000 samples/sec ?.

Encoding Information

Encoding describes the process of assigning representations to information

Digital IMages, Video

1-BIT can be used to encode "BLACK" or "WHITE" color.
e.g. an image of size 1024x1024 pixels (points) may be encoded using 1 bit per pixel ("1"~ black, "0"~ white). Such image size is - 1Megabit.

8-BIT Code may represent one of 256 GRAY LEVELS between black and white for each pixel in a DIGITAL IMAGE (1024x1024 pixels). Such image size is - **1MegaByte.**

32 -BIT Code may encode true color of a pixel in a DIGITAL IMAGE (1024x1024 pixels).
Such image size is - 4 **MegaBytes.**

Digital video is 60 Frames/ sec.

1 Frame of a true color digital image of size (1024x1024 pixels) requires 4 MegaBytes to store it.
Video Throughput: **240 MegaBytes/sec.**

How much memory do you need to store 1 HOUR of digital video?

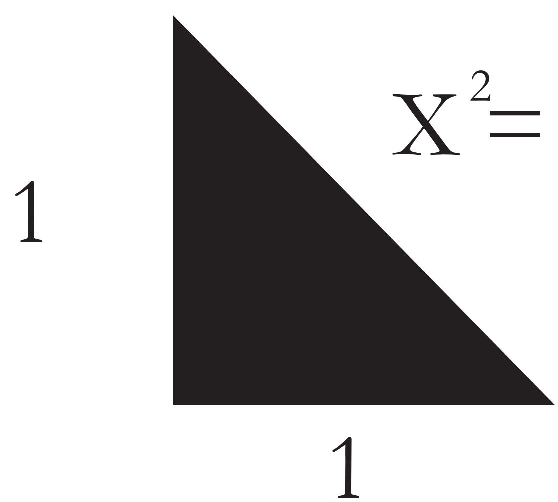
Encoding Information

Encoding numbers

IRRational Numbers can not be represented as a pair of Integers (m,n) and $n \neq 0$

How do we represent π ?

How do we solve the following problem on a computer?



$$X^2 = 1^2 + 1^2 \quad . \; X = \sqrt[2]{2}$$

Encoding Information

Encoding numbers

IRRational Numbers can not be represented as a pair of Integers (m,n) and $n \neq 0$

First 1001 digits of π

π =

3.14159265358979323846264338327950288419716939937 51
0582097494459230781640628620899862803482534211706798214 8
0865132823066470938446095505822317253594081284811174502 8
4102701938521105559644622948954930381964428810975665933 4
4612847564823378678316527120190914564856692346034861045 4
3266482133936072602491412737245870066063155881748815209 2
0962829254091715364367892590360011330530548820466521384 1
4695194151160943305727036575959195309218611738193261179 3
1051185480744623799627495673518857527248912279381830119 4
9129833673362440656643086021394946395224737190702179860 9
4370277053921717629317675238467481846766940513200056812 7
1452635608277857713427577896091736371787214684409012249 5
3430146549585371050792279689258923542019956112129021960 8
6403441815981362977477130996051870721134999999837297804 9
9510597317328160963185950244594553469083026425223082533 4
4685035261931188171010003137838752886587533208381420617 1
7766914730359825349042875546873115956286388235378759375 1
957781857780532171226806613001927876611195909216420198 9

In 2002 University of Tokyo team calculated 1, 241, 100, 000,000 digits. It took 602 hours and required more than 1Terabyte of memory on a HItachy SR8000 computer.

Assingnment No. 1A

1.

The purpose of this program is to understand and experience the limits of standard data types when dealing with irrational numbers. Each standard data type has its min and max values. Thus the number of digits representing irrational number is limited(precision is limited).

Write a program that computes and stores in memory any specific irrational number(choose any number you like) with a desired degree of precision.

For example, you can compute Pi with 10, 20, 100, 200, 300, 400 digits after the decimal point.

Measure and report the computation time.